

## Virtual Color Superconductivity and Nucleon Structure\*

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The observed electromagnetic (EM) properties of a nucleon at high energy are studied to search for a possible metastable color superconducting phase, called a virtual phase, that has an energy density close to the true ground state of the vacuum. The discussion is based on a mechanism of spontaneous partial breaking of the EM  $U(1)$  gauge symmetry inside a color superconductor. Favorable evidences for such a scenario are found.

**1. Introduction**

Theoretical and empirical studies seem to favor [1] the possibility that there exists a metastable color superconducting phase with its energy density close to the chiral symmetry breaking phase of the strong interaction vacuum. Since the color superconducting phase breaks the EM  $U(1)$  gauge symmetry spontaneously, it is quite likely that it could be found from the response of a nucleon to external EM probes. The questions are 1) why it is detectable 2) in which way it can be seen and 3) does it correspond to reality? In this talk, I would like to introduce a recent work done on this subject [2].

The problems concerning the EM properties of a nucleon that this work try to provide a logically coherent picture are the following

1. Possible violation of the GDH sum rule ( $\sim 10\%$ ), which is still not saturated by the data at the present [3].
2. The rapid rise of the structure functions  $F_2(x)$  and  $g_1(x)$  for a nucleon at small  $x$  measured at HERA and SMC. It seems to lead to a violation of Froissart bound.
3. The difference between  $F_2(x)$  extracted from charged lepton DIS and the one from the neutrino DIS.
4. And others [2].

Due to the length limit, I can only briefly discuss some of them and suppress most of the known references except those new ones. They can be found in Refs. [1,2]. The basic assumptions sufficient for the present talk are 1) there is a virtual color superconducting phase for the strong interaction vacuum and 2) together with the normal quasi-particles of the true vacuum phase, the quasi-particles of the virtual phase has a small probability to participate in the high energy processes. They can be substantiated by more solid theoretical argumentation [1], but we have no time for that here. Therefore a nucleon and any hadrons containing up and down quarks can become superconducting with a small probability [1]. This superconducting aspect of the

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nucleon is studied in more detail in the following. Our emphasis is on those properties of the color superconducting phase that are absent in the normal chiral symmetry breaking phase.

## 2. Partial Breaking of EM Gauge Symmetry

In a spontaneous breaking of a global symmetry, massless Goldstone bosons appear following the Goldstone theorem. What is less mentioned in literature is the separation of the corresponding charge that generates the symmetry. In a color superconductor, the EM vertex can be written as

$$J_{em}^\mu = J_{core}^\mu + \frac{l^\mu}{q \cdot l} J_{spread}, \quad (1)$$

where  $J_{core}^\mu$  is the contribution from the “core” part of the charge on the particle and the scalar function  $J_{spread}$  is the strength of the spreaded component of the charge that is carried away by the massless Goldstone boson.  $J_{spread}$  is related to the mass matrix that breaks the symmetry via a Ward identity. The 4-vector  $l^\mu$  is the longitudinal polarization of the Goldstone boson in a momentum carrying “medium”, like a nucleon, ( $l^\mu = q^\mu$  in the vacuum). Although  $q_\mu J_{em}^\mu = 0$  even in the symmetry breaking phase due to gauge invariance,  $J_{core}^\mu$ , which contribute to the observables in DIS [2], satisfies

$$q_\mu J_{core}^\mu = -J_{spread} \neq 0. \quad (2)$$

If the symmetry is local, like the EM  $U(1)$  gauge symmetry, then Higgs mechanism prevents the Goldstone boson to participate in physical collisions, which is well stated in text books. The effects of the Goldstone bosons are not visible in observables. For a color superconductor, it can be expressed as the following: the quark–quark EM interaction contains an additional term due to the exchange of a Goldstone boson which is absent in the symmetric phase. For example, the lowest quark–quark scattering T-matrix is of the following form

$$T = (iJ_{core})_\mu G_T^{\mu\nu} (iJ'_{core})_\nu + g^2 (iJ_{spread}) G_G (iJ'_{spread}) \quad (3)$$

where  $G_T^{\mu\nu}$  is the full propagator of a photon,  $g \sim O(1)$  is the coupling constant between the Goldstone boson and the quarks and  $G_G$  is the propagator of the Goldstone boson. Then Ward identities that specifies the right hand side of Eq. 2 and others can be used to recast Eq. 3 into the following form

$$T = (iJ_{core})_\mu (G_T^{\mu\nu} + G_L^{\mu\nu}) (iJ'_{core})_\nu \quad (4)$$

with the vacuum

$$G^{\mu\nu} = \frac{-g^{\mu\nu} + q^\mu q^\nu / m_\gamma^2}{q^2 - m_\gamma^2} \quad (5)$$

a propagator for a genuine massive vector particle that couples to the quark with the strength of the electromagnetism  $\alpha_{em} \sim 1/137$ . It should be noted that the scattering between two quarks generated by the Goldstone boson, like pions, is a strong interaction with strength  $g \sim 1$ . From this it can be concluded that  $G_T^{\mu\nu}$  also contains a component of strong interaction, which must cancel the  $g^2 G_G$  term. The fact that quark–quark interaction remains small in an EM interaction depends on a delicate cancellation of strong interaction effects that are responsible for the symmetry breaking.

Things are not that simple in the case of hadronic color superconductivity since the realistic order parameters for it does not break the EM  $U(1)$  gauge symmetry directly. It breaks the

global  $U(1)$  symmetry corresponding the nucleon number ( $1/3$  of the fermion number for the up and down quarks), which is itself a component of the baryon number, under the most natural scenario of color superconductivity [1]. It is induced by a condensation of diquarks made up of light up and down quarks. In this case, the Higgs mechanism of cancelling the effects of the Goldstone boson is effective only in the subspace of normal hadrons made up of up and down quarks. Other hadrons and all leptons do scatter with normal hadrons through exchange of the (or the lack of the) Goldstone bosons of the  $U(1)$  symmetry breaking connected to the nucleon number.

Let us consider the lowest order (in the EM coupling) lepton–nucleon scattering interested here, which can be written as

$$T(l + h \rightarrow l' + h') = (ij)_\mu G_T^{\mu\nu} (iJ'_{core})_\nu \quad (6)$$

with  $j_\mu$  the matrix element of the EM current operator and  $J'_\mu$  that of the hadrons (or up/down quarks). The Goldstone boson does not contribute to the lepton-quark scattering since leptons carry zero nucleon charge. Therefore, using the relation  $G_T^{\mu\nu} = G^{\mu\nu} - G_L^{\mu\nu}$  we have

$$T(l + h \rightarrow l' + h') = T_{em} + T_{strong} = (ij)_\mu G^{\mu\nu} (iJ'_{core})_\nu - (ij)_\mu G_L^{\mu\nu} (iJ'_{core})_\nu \quad (7)$$

where  $T_{em}$  is a small EM interaction scattering kernel corresponding to the first term on the right hand side of the above equation and the rest,  $T_{strong}$ , is an induced strong interaction kernel due to the lack of the Goldstone boson contributions between lepton and quarks in the color superconducting phase.

It implies that there is a significant strong interaction component in the semi-leptonic DIS that is absent in the normal hadron–hadron collision due to the Higgs cancellation in the later situations if a hadron has a “superconducting component” or if there is a close by virtual color superconducting phase for the strong interaction vacuum.

### 3. The Froissart Bound in DIS

The Froissart bound requires that the total cross section for a hadron–hadron collision at high enough energy must satisfies

$$\sigma_{tot}^{hh} \leq \text{const} \times \ln^2 s \quad (8)$$

with  $s$  the energy of the hadron involved. It turns out that the physical hadron–hadron total cross section do indeed satisfies the bound: in fact it lies just on the bound. In Regge theory, the reggeon exchanged that is responsible for the behavior is called the soft pomeron with an intercept of  $\alpha_P \propto 1.08$ .

The total cross section for a  $\gamma^* N$  interaction extracted from the semi-leptonic DIS appears to grow much faster than the right hand side of Eq. 8, in fact

$$\sigma^{\gamma^* N} \sim s^{0.4}, \quad (9)$$

which, in the language of the Regge theory, corresponds to an exchange of a hard pomeron with an intercept of  $\alpha_{P'} \approx 1.4$ . What cause such a higher rate of particle production remains much of a mystery from the unitarity point of view, albeit perturbative QCD can explain some of them, some problems remain [4]. It is not sure the spin content of a nucleon can be simultaneously explained [6].

The assumption that there is at least one color superconducting phase for the strong interaction vacuum state makes such a behavior very natural. Since the extra final states produced is due to the exchange of the (lack of) Goldstone boson of the color superconducting phase.

#### 4. Empirical evidences

The consequences of the assumption that a nucleon contains a superconducting companion which lives in the virtual color superconducting phase can be subjected to a variety of tests [1] using the known data. A partial list of them are 1) violation of GDH sum rule which is still not eliminated by the current data [3] 2) charge separation (Eq. 1) manifested in the difference [2] between the structure function  $F_2$  extracted from the charged lepton neutral current DIS and from the neutrino charged current DIS at small Bjorken  $x$  ( $x < 0.1$ ) 3) the appearance [2] of a photon-like vector pomeron coupled to a non-conserved current (Eq. 2) in the meson production processes of a NN collision 4) the violation of the Froissart bound (Eq. 9) discussed above and the more detailed predictions concerning the different energy dependences of the electro-production of the  $\rho$ ,  $\phi$  and  $J/\psi$  mesons under the hard pomeron hypothesis [5]. They are all in qualitative agreement with the data. Under the hard pomeron hypothesis, the small  $x$  behavior of the polarized structure functions  $g_1$  and  $g_2$  of a nucleon have the following small but quite singular component at small  $x$  before the Bjorken limit

$$g_1(x) \sim \frac{1}{x} F_2(x) \sim x^{-1.4}, \quad g_2(x) \sim \frac{1}{x^2} F_2(x) \sim x^{-2.4} \quad (10)$$

which comes from the superconducting component of a nucleon mentioned above. The first equation above is consistent [2] with the current SMC data that has its smallest  $x \sim 10^{-4}$ . Such a behavior is hard to explain in a perturbative QCD scheme.

Albeit many evidences of quite different origin are correlated to point to the possibility that there is at least one close by virtual color superconducting phase for the strong interaction vacuum, more work are needed to firmly establish it [1].

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